

**INTERNATIONAL PORTFOLIO DIVERSIFICATION: AN EVIDENCE FROM  
DEVELOPED, EMERGING AND FRONTIER MARKETS**

**Muhammad Ayaz Khan**

Department of Management Sciences

COMSATS University Park Road,

Islamabad, 45550

Pakistan

mohdkhanayaz@gmail.com

**Dr. Faheem Aslam\***

Department of Management Sciences

COMSATS University Park Road,

Islamabad, 45550

Pakistan

[faheem.aslam@comsats.edu.pk](mailto:faheem.aslam@comsats.edu.pk)

**Wahbeeah Mohti**

Department of Business Administration,

Iqra University Islamabad Campus, Pakistan

wahbeeah.mohti@iqraisb.edu.pk

**ABSTRACT**

The purpose of this study is to investigate the wholesome benefits of diversification and the resulting investment opportunities observe by the frontier equity markets to international portfolio investors. Secondly, the study observes if international portfolio diversification states any optimal risk and return trade-off benefits in short-selling or without it. The study is based on daily closing prices data of 50 stock market indices from MSCI market classification throughout 2005 to 2020. The distinct combination of weighted portfolios from emerging, developing, and frontier markets are introduced. The results confirm that efficient frontier markets are negatively correlated to the main developed and developing countries, offer optimal risk, and return combinations. These results are consistent both in the case of short selling and without short selling. Such findings might be helpful to construct optimal portfolios as well as empirical applications in investment theory.

**Keywords:** *Efficient Frontier, Diversification. Minimum variance portfolio*

## Introduction

Whenever an investment is considered, the two main investment components are also measured, risk and returns. The ultimate objective of an investor is to achieve maximum return for his investment (Williams 1938). Investor also desires minimum disaster with the return, according to safety first principle by (Roy, 1952). For this purpose, investors prefer to use the most reliable and famous investment techniques in modern finance, such as portfolio diversification (Markowitz, 1952). The reduction of risk related to a specific firm by diversification is apodictic. It is also accepted that risk related to specific economy can be reduced by diversifying the investment internationally. Global diversification is much easier recently with improved financial liberalization between countries, technology, and sharing of information. Despite the improvements and great diversification opportunities offer by global investments, investors still prefer to invest domestically, which we may call home-bias.

There is a bundle of literature regarding international diversification benefits (Ahmed, Ali, Ejaz, & Ahmad, 2018; Elsayed, Nasreen, & Tiwari, 2020; Fletcher, 2021; Oloko, 2018; Shahzad, Bouri, Roubaud, & Kristoufek, 2020; VO, 2017). Several methodologies and techniques are used on a different range of data in the literature. The evidence of correlation and integration between markets is also found in the literature. From the pioneer study of Grubel (1968), the below arrangement of literature is evidence for the benefits of international diversification. Other studies, such as Levy and Sarnat (1970), find benefits of diversification for American investors discourages domestic investment. Likewise, French suggest that a low correlation between developed markets indicates diversification benefits. Altay-Salih, Muradoglu, and Mercan (2002), Conover, Jensen, and Johnson (2002) also find similar results through their study. Some efforts claim that international diversification is beneficial for developed countries investors and for frontier and emerging markets investors. Frontier markets are pre-emerging markets, which are less accessible in terms of information, for investment and the economic and financial condition of the belonging country may not be good as compare to emerging and developed markets. Studies such as Li, Sarkar, and Wang (2003), Driessen and Laeven (2007) also consider short sell constraints and find various results. Short selling is a type of investment transaction in which the selling of stocks occurs, but the seller does not own the stocks. This type of trade is a short sale (Elton, Gruber, Brown, & Goetzmann, 2009). Short selling is prohibited in some countries, and it may not be practiced due to fewer lenders or high lending fees (Bris, Goetzmann, & Zhu, 2007). Najeeb, Bacha, and Masih (2015) find that the correlation among sample markets increase over time and not beneficial for diversification. This increasing correlation provides an opportunity to look less developed countries for diversification. Berger, Pukthuanthong, and Yang (2011) also confirm that frontier markets are less integrated with developed markets. Thus, they may provide international diversification benefits when they appear in a diversified portfolio that includes developed and emerging equities. Some studies also show the capital flow to frontiers markets, such as Montiel (1994), which shows evidence of openness of developing markets for capital flow. Capital tends to flow more towards countries with higher productivity growth and higher investment (Gourinchas & Jeanne, 2013). Mostafa and Stavroyiannis (2016) documented the correlations within BRIC and with developed markets

and observed low correlation, thus concluding that this is diversification benefits an internationally diversified portfolio. Sukumaran, Gupta, and Jithendranathan (2015) find that diversifying into frontier markets is beneficial for both Australian and US investors but the benefit to the US investor than Australian investors. Balçılar, Demirer, and Hammoudeh (2015) find the international diversification benefits in the GCC (Gulf Cooperation Council) markets. Thereby, it can be surmise and postulate that including frontier markets in individual international portfolio or exchange trade funds (ETFs), mutual funds will be lucrative for investors. From a different perspective, Neu-Ner and Firer highlighted the benefits of diversification in reducing the unique risk of portfolios of randomly selected Johannesburg stock exchange (JSE) shares. They found that 90% of the benefits of diversification may be obtained by holding a random portfolio of only 30 shares. In a recent study, Lim and Ong (2021) suggested Shape-Based Clustering as an alternative portfolio management tool. Likewise, Fletcher (2021) used different simulations and documented that US international equity closed-end funds (CEF) provide out-of-sample diversification benefits. From a different perspective, Elsayed et al. (2020) proved that addition of energy products with stock market gives and optimal portfolio and heavily weighted to the stock markets.

Literature endorses that the expected returns and risk of portfolios mainly depend on the correlation between stock returns. The literature also indicates a low correlation between developed and frontier markets. The low correlation between both classes of markets is due to the difference in economic and financial structure. Since the frontier market is less developed and offers considerable diversification benefits, this study will analyze Morgan Stanley composite index (MSCI)<sup>1</sup> classified markets for the potential diversification benefits by mean-variance analysis. This research examines the performance of MSCI class indices before and after adding frontier and regional frontier stock markets in developed countries Indies. The study will quantify the optimal weights for markets in the global minimum variance portfolio and contribute efficient frontier graphs to the literature. Since frontier markets are the latest discovery in the stock market classes, the diversification benefits offered by these are the most attractive issues to study for the investor. Several mutual funds, management companies, and institutional and retail investors invest in index funds or equities to replicate indices' movements instead of individual securities. This study would have multiple implications for international investors to diversify their investments by adding the non-correlated markets in the portfolio.

This paper advances the benefits and opportunities of international diversification in several ways after confirmation of affirmative findings of prior efforts. We will construct two types of efficient frontiers containing 100 optimal portfolios. Firstly, portfolios of MSCI classified developed markets. Secondly, construction of the portfolios of developed with emerging markets and developed with frontier markets index and then compares both portfolios. We further analyze regional frontier markets to examine the advantage of diversification benefits

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<sup>1</sup>MSCI Inc. is a leading provider of investment decision support tools to investors globally, including asset managers, banks, hedge funds and pension funds. MSCI products and services include indexes, portfolio risk and performance analytics, and ESG data and research. The index is divided into three categories of frontier, developing and developed countries.

due to geographically divided markets. The frontier markets are less correlated and geographically diversified with the developed market; these sets are a motivating market class to add to the portfolio for diversification. Studies of the frontier market for diversification are not very common because the markets are recently available for investments. We will estimate the magnitude of the change in the returns and risk of the global minimum variance portfolio due to frontier market addition. We observed the diversification benefit due to the inclusion of frontier markets into already diversified developed markets portfolios. This study can be distinct in three ways. Firstly, it constructs eleven different portfolios with short selling and without short selling, &with the combination of three different types of markets. The construction aims to compare all portfolios in terms of risk and returns, so it will help investors to achieve desire benefits from international portfolio investments. Secondly, a graphical comparison of efficient frontiers consists of 100 optimal portfolios of different market combinations. Thirdly, the study quantifies different weights to markets in a global minimum variance portfolio.

### Data & Methodology

An efficient frontier is a curve consisting of portfolios between the global minimum variance portfolio and the maximum return portfolio (Elton et al., 2009). The set of portfolios on a curve or efficient portfolios gives maximum returns at a given variance of return or minimum variance at a given level of rate of returns (Reilly & Brown, 2011). In this study, these efficient portfolios consist of different sizes, such as developed and emerging market portfolios contains eighteen markets each, and frontier markets consist of fourteen markets. Details of all other portfolios are shown in table 1. We combined 50 markets in several ways from the different classifications of MSCI and constructed eleven portfolios.

**Table 1: Portfolios and their composition:**

<b>Portfolios</b>	<b>Combination of markets</b>
Developed markets portfolio	18 Developed markets
Emerging Markets Portfolio	18 Emerging markets
Frontier Markets Portfolio	14 Frontier markets
Developed with Frontier markets Portfolio	18 Developed markets and 14 Frontier markets
Developed with Emerging Markets Portfolio	18 Developed and 18 Emerging markets
Emerging Markets with Frontier Markets Portfolio	18 Emerging markets and 14 Frontier markets
Developed with Asian Frontier Markets Portfolio	18 Developed markets and 3 Asian Frontier markets
Developed with Middle East Frontier	18 Developed markets and 3 Middle East Frontier

Market Portfolio	markets
Developed with African Frontier Market Portfolio	18 Developed markets with 3 African Frontier markets
Developed with European Frontier Markets Portfolio	18 Developed and 3 European Frontier markets
Global Markets Portfolio	18 Developed Markets, 18 Emerging markets, and 14 Frontier markets

The daily prices of stock indices are collected from investing.com, while the market classification is available at MSCI<sup>2</sup>. The final data selected for the study is daily data from 1 January 2005 to 31 December 2020 for 50 countries each. This study estimates daily returns for markets in equation 1. The arithmetic means of return in equation 2 and weights obtained by optimization with Lagrange multiplier in equation 3 estimates the expected return of global minimum variance portfolio in equation 7. Similarly, the standard deviation of markets and equation 6, the covariance between markets, and equation 3 are used to measure the risk of the global minimum variance portfolio. Equation 9 and 10 indicates the diversifiable and non-diversifiable risk in the portfolio. Equation 4 is constraints that indicate the sum of all weights in the portfolio should be 1. Equation 5 shows the restriction of short selling, but it can be relaxed when short selling is allowed as in this study.

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

Where,  $R_t$  is return at time " $t$ " which is return of the current day.  $P_t$  is daily closing price at time " $t$ " which is closing price of current day.  $P_{t-1}$  is daily closing price at " $t-1$ " which closing price of previous day.

$$\overline{R_{AM}} = \frac{1}{n} \sum_{i=1}^n R_i \quad (2)$$

Where,  $\overline{R_{AM}}$  is Arithmetic mean return,  $n$  is number of markets returns and  $\sum_{i=1}^n R_i$  is sum of  $n$  number of markets returns.

$$W_{gmv} = \frac{COV^{-1} + 1}{1 + COV^{-1}} \quad (3)$$

<sup>2</sup><https://www.msci.com/market-classification>

Where,  $W_{\text{gmv}}$  is weights of markets in global minimum variance portfolio and COV is inverse of the variance-covariance matrix of returns.

### Constraints

$$\sum_{i=1}^n W_i = 1 \quad (4)$$

$$W_i > 1 \quad (5)$$

Where,  $W_i$  is weights for stock "i" in the portfolio.

### COVARIANCE BETWEEN MARKETS

The diversifiable risk is due to the covariance between markets:

$$\sigma_{ij} = \sum (R_i - \bar{R}_i)(R_j - \bar{R}_j) \quad (6)$$

Where,  $\sigma_{ij}$  is the covariance between assets "i" and "j",  $R_i$  is the expected return of market "i",  $R_j$  is the expected return of market "j",  $\bar{R}_i$  is mean return of market "i", and  $\bar{R}_j$  is mean return of market "j"

Many studies indicate that the correlation between securities is inversely related to the risk or benefit of diversification (Gilmore & McManus, 2002), (Haque, Hassan, & Varela, 2001). There are strong correlations critiques because of the rise and decline of correlation during a crisis, especially when the portfolio is internationally diversified. It is said that the correlation trend towards declining in bull markets and increasing when the market is bear (Lin, Engle, & Ito, 1994) and (Longin & Solnik, 2001). In his study, this criticism is very well defended by (Asness, Israelov, & Liew, 2011). He claims that international diversification may not be beneficial during a crisis or short term, but it is beneficial for the long term.

### RETURN FOR PORTFOLIO

The formula for the expected return of global minimum variance portfolio:

The expected return  $E(R_p)$  of the global minimum variance portfolio is

$$E[R_p] = \sum_{i=1}^N W_i E[R_i] \quad (7)$$

Where,  $E[R_p]$  is the expected return on the portfolio, "p",  $N$  is the number of stocks in the portfolio "p",  $W_i$  is the proportion of the portfolio "p" invested in stock "i" and  $E[R_i]$  is the expected return on stock "i"

## RISK FOR PORTFOLIO

The variance of returns is considered as risk of the markets.

And its return variance ( $\sigma_p^2$ ) is

$$\sigma_p^2 = \sum_{i=1}^n W_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{i \neq j}^n W_i W_j \sigma_{ij} \quad (8)$$

Where,  $\sigma_p^2$  is the risk of portfolio "p",  $\sigma_i^2$  is the variance on asset "i",  $\sigma_{ij}$  is the covariance between assets "i" and "j",  $W_i$  is the proportion of the portfolio "p" invested in stock "i" and  $W_j$  is the proportion of the portfolio "invested in stock "j"

Risk can be defined as the inconsistency in the expected returns. According to Grundy, risk differs from actual investment reward from expected reward. There are two types of risk, systematic and unsystematic, related to investment, as shown in equations 9 and 10. Systematic risk, also called "market risk" or "un-diversifiable risk," is directly related to overall movements in the general market and is due to broad macro factors affecting all assets in the portfolio. According to Cox and Griepentrog (1988) systematic risk deals with volatility, the changes in the price of stocks. Systematic risk remains with the investments after the combination of different assets to an efficient portfolio (Jacob, 1971). Unsystematic risk is also termed as "specific risk", "diversifiable risk" or "residual risk" related to a particular stock or asset in the portfolio and attributable to factors unique to stock in the portfolio. All residual (or unsystematic) risk can be diversified away through diversification (Cox & Griepentrog, 1988). The mathematical representation of risk is variance, firstly use by (Markowitz, 1952). Later, many studies use variance as risk, such as (Newbould & Poon, 1993), (Evans & Archer, 1968), (Statman, 1987) and (Elton & Gruber, 1977). It is not necessary that the portfolio that gives maximum return will provide minimum risk. There is a rate at which an investor can get a maximum return by taking a risk or minimum return by avoiding risk (Markowitz, 1952).

### NON-DIVERSIFIABLE RISK

$$\sum_{i=1}^n W_i^2 \sigma_i^2 \quad (9)$$

### DIVERSIFIABLE RISK

$$\sum_{i=1}^n \sum_{j=1, i \neq j}^n W_i W_j \sigma_{ij} \quad (10)$$

Where,  $\sigma_i^2$  is the variance on asset "i",  $\sigma_{ij}$  is the covariance between assets "i" and "j",  $W_i$  is the proportion of the portfolio invested in stock "i" and  $W_j$  is the proportion of the portfolio invested in stock "j"

The study estimates daily returns for markets in equation 1. The arithmetic means of return in equation 2 and weights obtained by optimization with Lagrange multiplier in equation 3 estimates the expected return of global minimum variance portfolio in equation 7. Similarly, the standard deviation of markets and equation 6, the covariance between markets, and equation 3 are used to measure the risk of the global minimum variance portfolio. Equation 9 and 10 indicates the diversifiable and non-diversifiable risk in the portfolio. Equation 4 is constraints that indicate the sum of all weights in the portfolio should be 1. Equation 5 shows the restriction of short selling, but it can be relaxed when short selling is allowed, as in this study.

### Empirical Findings

In table 2, we report the risk and return of all global minimum variance portfolios. By comparing the risk of the portfolio with our benchmark portfolio, which is the Global markets portfolio. The global market portfolio consists of 50 markets around the world. We can observe that few arranged portfolios beat the return of benchmark portfolios at higher risk, but none of the portfolio risks is below the risk of benchmark in both conditions of short selling. With short-selling constraints, the closest risk is developed with frontier market risk, which is 0.434%. If short selling is allowed, the closest risk is again developed with frontier markets portfolio risk of 0.043%. On the other column, the return of many markets outperforms, but their risk is also very high, which means these markets generate a high return with high risk. In other words, we can say these markets are suitable for risk lovers. Investing in developed with frontier market portfolio gives us 0.033% returns at 0.434% risk when short selling is not allowed, which is 0.013% more return than 0.020% return of developed markets. The risk is also deteriorating from 0.895% to 0.434%. There is a 0.015% improvement in return if a short sale is allowed and a 0.41% decline in risk. The return we can get from developed with emerging markets portfolio is 0.007% higher than developed markets portfolio if short sale not and the risk is decline to 0.694% from 0.895%. If the constraint is relaxed to allow short sales, the developed portfolio with emerging markets gives a 0.006% greater return than the developed markets portfolio with 0.177% less risk. If we compare the developed markets portfolio with the portfolio which contains emerging and frontier markets. The emerging with frontier markets portfolio gives 0.017% higher return at 0.448% lower risk with short-selling constraints. When short selling is allowed, the portfolio consisting of emerging with frontier markets shows lesser improvement of 0.009% compared to portfolios with short selling restriction at 0.390% risk. The return can be improved to 0.049%, and the risk can also be reduced to 0.0619% by adding Asian frontier markets in the portfolio if short selling is not allowed.

**Table 2: Risk and Returns at global minimum variance portfolios**

<b>Portfolios</b>	<b>Return</b>	<b>Risk</b>
<b>Without Short Selling</b>		
Developed markets portfolio	0.02	0.895
Emerging Markets Portfolio	0.036	1.106
Frontier Markets Portfolio	0.036	0.492
Developed with Frontier markets Portfolio	0.033	0.434
Developed with Emerging Markets Portfolio	0.027	0.694
Emerging Markets with Frontier Markets Portfolio	0.037	0.447
Developed with Asian Frontier Markets Portfolio	0.049	0.619
Developed with Middle East Frontier Market Portfolio	0.015	0.631
Developed with African Frontier Market Portfolio	0.034	0.554
Developed with European Frontier Markets Portfolio	0.012	0.695
Global Markets Portfolio	0.034	0.402
<b>With Short Selling</b>		
Developed markets portfolio	0.028	0.836
Emerging Markets Portfolio	0.036	1.104
Frontier Markets Portfolio	0.037	0.491
Developed with Frontier markets Portfolio	0.035	0.423
Developed with Emerging Markets Portfolio	0.033	0.659
Emerging Markets with Frontier Markets Portfolio	0.037	0.446
Developed with Asian Frontier Markets Portfolio	0.051	0.598
Developed with Middle East Frontier Market Portfolio	0.019	0.606
Developed with African Frontier Market Portfolio	0.038	0.536
Developed with European Frontier Markets Portfolio	0.017	0.663
Global Markets Portfolio	0.036	0.391

When short-selling is allowed, the return is increased to 0.051% from 0.028%. When Asian frontier markets are included with developed markets portfolios, a decline in risk from 0.836% to 0.598% is also observed. The restriction of short selling and the addition of Middle East frontier markets in the portfolio decreases the returns to 0.015% from 0.020%. When Middle East frontier markets were added to developed markets portfolios and short selling restrictions, the returns declined 0.009% from 0.028% to 0.019%. With or without short selling, the risk for both conditions deteriorates from 0.895% to 0.631% when short selling is not allowed and declines to 0.606% from 0.836% when short selling is allowed. A short-selling constraint portfolio of developed and African frontier markets offers a 0.014% excess return to the developed market portfolio. The portfolio also reduces risk by 0.341%. If short selling is allowed, the addition of African frontier markets to developed markets portfolios provides 0.010% additional benefits in terms of return and also decreases the risk by 0.300%. The inclusion of European frontier markets reduces return of portfolio from 0.020% to 0.012% if short selling is not allowed. The risk is also reduced from 0.895% to 0.695%, which is beneficial for the investor. Likewise, when short selling is allowed, the portfolio gives a 0.017% return which is 0.011% less than the 0.028% return of developed markets portfolio. The risk also decreases from 0.836% to 0.663%.

Similarly, this study first plots the efficient frontier for the developed markets and portfolio containing developed with emerging and developed with frontier markets. All frontiers represent 100 different portfolios; each starts from a global minimum variance portfolio but ends at higher risk and returns. The study also plots the global market portfolio frontier and arranged it on the same figure for comparison purposes. We compare efficient frontiers of our benchmark portfolio with other efficient frontiers, as shown in Figure 1 and Figure 2. The benchmark frontiers are closer to y-axis, which means low risk. These benchmark frontiers are not very high vertically, which means investors can get low returns by investing in the benchmark compared to others. The slope of all efficient frontiers is very high when short selling is allowed, which means the rate increasing risk of the portfolio is higher than the rate of return.

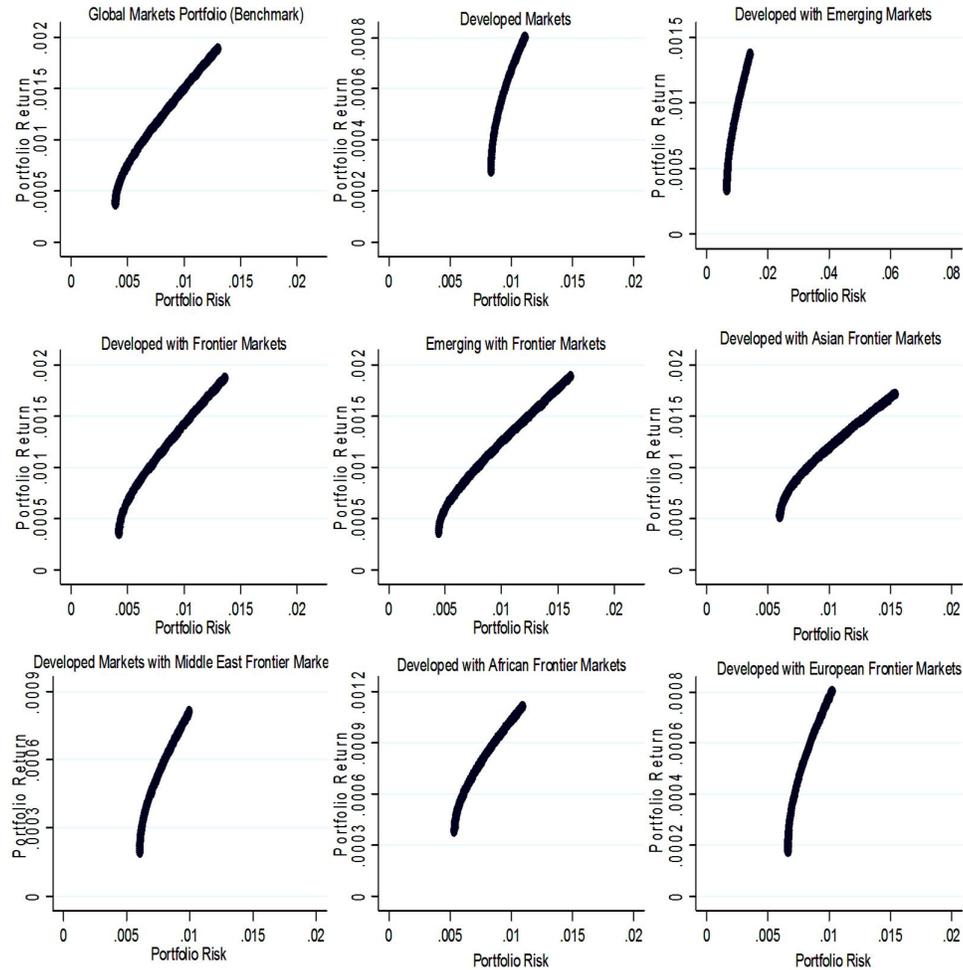
Meanwhile, when short selling is not allowed, the slope of the efficient frontiers is low. In other words, the frontiers trend shows more horizontally long than vertically, which means the portfolio returns increase slower than the risk of the portfolio. The frontiers for developed with frontier markets portfolios are closer to the vertical axis than developed markets portfolios, which means these portfolios are less risky than the portfolio containing developed markets with both conditions of short selling. The developed market frontier starts from the global minimum variance portfolio and ends at 0.08% returns with 1.2% risk. The frontier for a portfolio containing developed and emerging markets ends at 0.06% with 1.6%. Similarly, the frontiers with short selling constraints also start from the global minimum variance portfolio and end at a portfolio with higher risk and returns. The efficient frontiers for emerging with frontier markets portfolios show the investment is less risky and more beneficial than developed markets portfolio, with both conditions of short selling. Comparing emerging markets with efficient frontiers consisting of emerging and frontier markets

portfolios shows a slight shift to the upside. It means no significant outcomes due to the inclusion of frontier markets in a portfolio containing emerging markets in terms of returns. The left side movement of the curve at the start shows the risk reduction. The shift of efficient frontiers of developed with Asian frontier markets frontier to the left side and up vertically means improvement in returns and decline in risk when short selling is allowed.

Similarly, efficient frontiers with short selling restrictions, the shift of curve from right to left, and moving vertically up shows the decline in risk and return improvement. The frontier for portfolios consists of developed and the Middle East frontier markets near the y-axis and drag towards the x-axis. It means portfolios by developed and Middle East frontier markets are less risk at lower returns. Similarly, the efficient frontier without short selling also shows that portfolios' investments, including developed and the Middle East frontier markets, are less risky at lower returns. Without short-selling constraints for portfolios, the frontier consists of developed and African frontier markets that are shifted closer to the y-axis and higher than the frontier of developed markets. This means the minimum variance portfolio on developed with African markets frontier is more beneficial than the minimum variance portfolio on developed markets frontier.

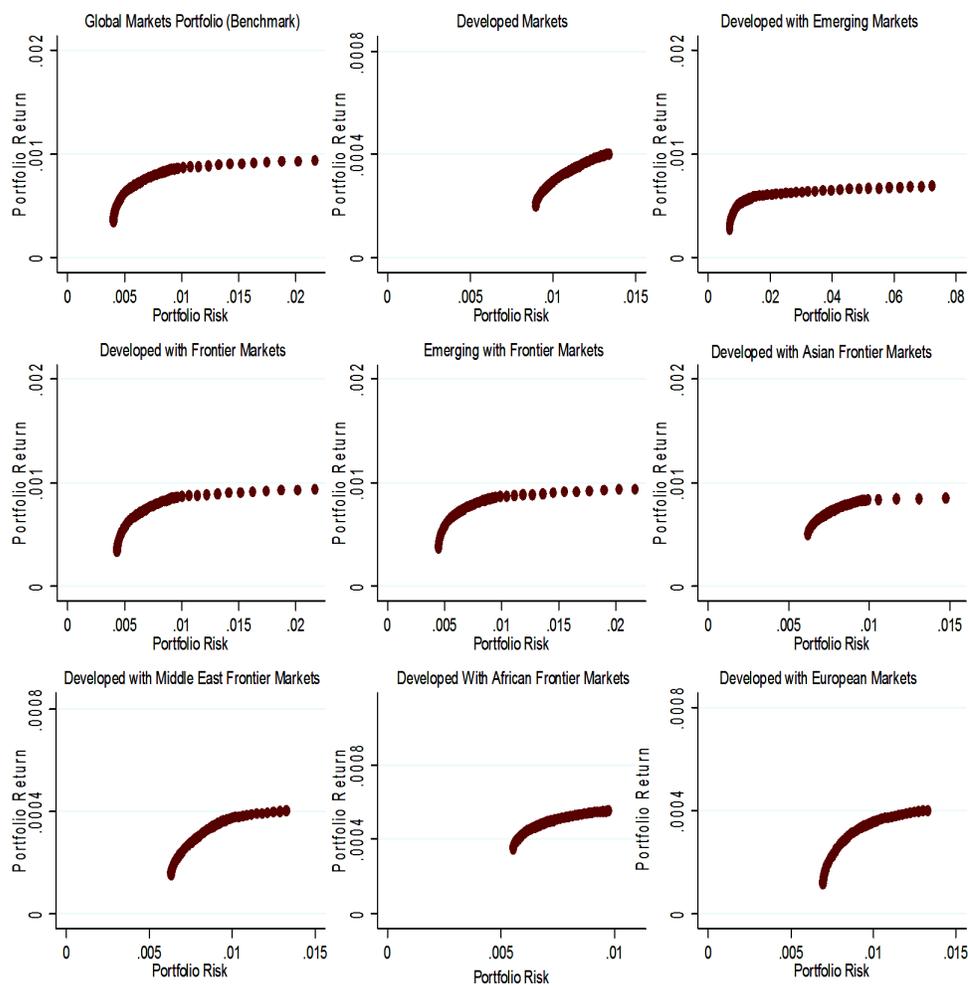
Similarly, the frontier curve developed with African frontier markets is shifted close to the y axis and away from the x-axis. This means the portfolios containing developed and frontier markets provide more return and are also less risky. The curve for developed with European frontier markets is close to the y-axis and x-axis, which means the portfolio of developed with European markets is less risky with low return and short selling. With short restrictions, the developed and European frontier markets curve is closer to portfolio return and risk axes. This means the developed with European market portfolio gives return and less risky, which is suitable for risk-averse investors.

Figure 1 exhibits two graphs for efficient frontiers without short-sale constraints, combined side by side to compare and analyze. The two frontiers represent 100 portfolios containing developed markets, and 100 portfolios are developed with European frontier markets. The curve for developed with European frontier markets on the right side is close to the y-axis and x-axis, which means the portfolio developed with European markets is less risky with low return. The frontier curve trend and shape are very similar—similarly, figure 2 exhibits the efficient frontier with short sale restriction. Likewise, in figure 1, the curves in figure 2 also show similar trends and shapes, but the frontier length for developed with European frontier markets is longer. It means that investors investing in portfolios containing developed with European frontier markets have a large range of risk and return combinations.



**Figure 1: Efficient frontier for comparison if short selling is allowed.**

The observed portfolio shows different combinations of weights to all three classes of markets for both conditions of short selling. Table 3 shows the arrangement of weights for all types of markets at all observed global minimum variance portfolios. One common finding we can observe in table 3 regarding frontier market weights is that almost all portfolios allocate a large portion to frontier markets. Like, in the Global market portfolio, which gives the lowest risky portfolio, 67.28% weight is given to frontier markets when short selling is not allowed. As discussed above, the developed with frontier markets portfolio is the lowest risk portfolio after benchmark portfolio also allocates a large portion of weights to frontier markets, which is 77.31%.



**Figure 2: Efficient frontier for comparison if short selling is not allowed.**

**Table 3: Weights at global minimum variance portfolio**

Portfolios	Weights		
	Developed	Emerging	Frontier
<b>Without Short Selling</b>			
Developed markets portfolio	100	0.00	0.00
Emerging Markets Portfolio	0.00	100	0.00
Frontier Markets Portfolio	0.00	0.00	100
Developed with Frontier markets Portfolio	22.69	0.00	77.31
Developed with Emerging Markets Portfolio	60.61	39.39	0.00
Emerging Markets with Frontier Markets Portfolio	0.00	16.40	83.60

Developed with Asian Frontier Markets Portfolio	48.34	0.00	51.67
Developed with Middle East Frontier Market Portfolio	50.30	0.00	49.70
Developed with African Frontier Market Portfolio	37.02	0.00	62.99
Developed with European Frontier Markets Portfolio	57.61	0.00	42.39
Global Markets Portfolio	19.55	13.17	67.28

### **With Short Selling**

Developed markets portfolio	100	0.00	0.00
Emerging Markets Portfolio	0.00	100	0.00
Frontier Markets Portfolio	0.00	0.00	100
Developed with Frontier markets Portfolio	25.22	0.00	74.78
Developed with Emerging Markets Portfolio	63.44	36.56	0.00
Emerging Markets with Frontier Markets Portfolio	0.00	16.25	83.75
Developed with Asian Frontier Markets Portfolio	51.54	0.00	48.45
Developed with Middle East Frontier Market Portfolio	53.43	0.00	46.56
Developed with African Frontier Market Portfolio	40.21	0.00	59.80
Developed with European Frontier Markets Portfolio	60.74	0.00	39.23
Global Markets Portfolio	22.01	12.95	65.04

The highest weight given to frontier markets is 83.60% in emerging with frontier markets, followed by 77.31% in developed with frontier markets. The lowest weight given to frontier markets is 42.39% in developed with European frontier markets. If short selling is allowed, the largest weight portion given to frontier is 83.75% in emerging with frontier markets portfolio. Again, the second-largest weight portion is 74.78% in developed with the frontier market portfolio, which is the best portfolio after benchmark in terms of riskiness.

### **Conclusions and Recommendations**

To reflect the interest of investors and researchers, this study is done to analyze the benefits that an investor can get by investing globally. The findings in this study are based on extensive, reliable daily closing prices data of 50 stock market indexes from MSCI market classification. Eleven portfolios are assembled using developed, emerging, and frontier markets by allowing short sales and not allowing short sales. This study discovered that

diversification opportunities could be improved by including emerging and frontier markets in the portfolio of developed markets. These findings agree with previous literature exploring the diversification benefits across different classes of markets, across regions (Berger et al., 2011). Investors who invested in a portfolio containing all three classes of markets or combined these three classes had beat the portfolios concentrated only on developed markets. The risk of a developed market portfolio is easily outperformed by the risk of all other markets' portfolios except the emerging markets portfolio. In other words, we can say that investors investing internationally in a portfolio containing frontier markets noticeably achieved higher benefits than investing only in a developed market if short selling is not allowed. Likewise, when short selling is allowed, the developed market portfolio is again beaten by all other globally diversified portfolios except emerging markets. (Jayasuriya & Shambora, 2009) also noted similar results that portfolios containing less correlated markets like frontier are more beneficial than developed market portfolios. The study also notices that a more significant portion of the global minimum variance portfolio is given to the frontier market.

The weight concentration towards frontier markets also indicates low covariance with other markets. The visual examination of the portfolio also exhibits that most of the efficient frontiers for the globally diversified portfolio are closer to the y-axis, which means low risk for the portfolios. Finally, it is also observed that our benchmark global markets portfolio remains unbeaten in terms of risk, with the support of prior literature such as (Driessen & Laeven, 2007), (Conover et al., 2002), and others. The researcher feels confident that frontier markets offer considerable diversification benefits with/without short selling. This study has significant risk management implications for investors who are still fearful of international diversification in less developed markets. The proposed combination of markets has presented that portfolio managers can find financially meaningful investable portfolios. Our outcome recommends that adding frontier markets may evidence to be sufficient for reaching optimality. Empirical results of this study can have important practical uses and applications for financial institutions, risk managers, portfolio managers, and financial regulators who desire to invest in stock indices. The study believe that the proposed combination of markets has the potential of producing realistic risk-return profiles and could potentially create better investable portfolios for portfolio managers globally. The analysis of the efficient portfolio frontiers, however, suggests a possible new trend in international portfolio diversification that would include relatively more frontier markets in the portfolio. This study is done with a purpose and fulfills all ethical value of research. The researcher is confident about our findings and believes that these findings will helpful practitioners well as academic researchers. The researcher also believed that further studies are required to discuss asset pricing issue in frontier markets.

## REFERENCES

- Ahmed, A., Ali, R., Ejaz, A., & Ahmad, I. (2018). Sectoral integration and investment diversification opportunities: evidence from Colombo Stock Exchange. *Entrepreneurship and Sustainability Issues*, 5(3), 514-527.
- Altay-Salih, A., Muradoglu, G., & Mercan, M. (2002). Performance of the efficient frontier in an emerging market setting. *Applied Economics Letters*, 9(3), 177-183.
- Asness, C. S., Israelov, R., & Liew, J. M. (2011). International diversification works (eventually). *Financial Analysts Journal*, 67(3), 24-38.
- Balcilar, M., Demirer, R., & Hammoudeh, S. (2015). Regional and global spillovers and diversification opportunities in the GCC equity sectors. *Emerging Markets Review*, 24, 160-187.
- Berger, D., Pukthuanthong, K., & Yang, J. J. (2011). International diversification with frontier markets. *Journal of Financial Economics*, 101(1), 227-242.
- Bris, A., Goetzmann, W. N., & Zhu, N. (2007). Efficiency and the bear: Short sales and markets around the world. *The journal of finance*, 62(3), 1029-1079.
- Conover, C. M., Jensen, G. R., & Johnson, R. R. (2002). Emerging markets: When are they worth it? *Financial Analysts Journal*, 58(2), 86-95.
- Cox, L. A., & Griepentrog, G. L. (1988). Systematic Risk, Unsystematic Risk, and Property-Liability Rate Regulation. *The Journal of Risk and Insurance*, 55(4), 606-627. doi:10.2307/253140
- Driessen, J., & Laeven, L. (2007). International portfolio diversification benefits: Cross-country evidence from a local perspective. *Journal of Banking & Finance*, 31(6), 1693-1712.
- Elsayed, A. H., Nasreen, S., & Tiwari, A. K. (2020). Time-varying co-movements between energy market and global financial markets: Implication for portfolio diversification and hedging strategies. *Energy Economics*, 90, 104847.
- Elton, E. J., & Gruber, M. J. (1977). Risk reduction and portfolio size: An analytical solution. *The Journal of Business*, 50(4), 415-437.
- Elton, E. J., Gruber, M. J., Brown, S. J., & Goetzmann, W. N. (2009). *Modern portfolio theory and investment analysis*: John Wiley & Sons.
- Evans, J. L., & Archer, S. H. (1968). Diversification and the reduction of dispersion: an empirical analysis. *The journal of finance*, 23(5), 761-767.
- Fletcher, J. (2021). Exploring the diversification benefits of US international equity closed-end funds. *Financial Markets and Portfolio Management*.
- French, K. R., & Poterba, J. M. (1991). *Investor diversification and international equity markets*. Retrieved from

- Gilmore, C. G., & McManus, G. M. (2002). International portfolio diversification: US and Central European equity markets. *Emerging Markets Review*, 3(1), 69-83.
- Gourinchas, P.-O., & Jeanne, O. (2013). Capital flows to developing countries: The allocation puzzle. *The Review of Economic Studies*, 80(4), 1484-1515.
- Grubel, H. G. (1968). Internationally diversified portfolios: welfare gains and capital flows. *The American Economic Review*, 58(5), 1299-1314.
- Grundy, K., & Malkiel, B. G. (1996). Reports of beta's death have been greatly exaggerated. *The Journal of Portfolio Management*, 22(3), 36-44.
- Haque, M., Hassan, M. K., & Varela, O. (2001). Stability, volatility, risk premiums, and predictability in Latin American emerging stock markets. *Quarterly Journal of Business and Economics*, 23-44.
- Jacob, N. L. (1971). The Measurement of Systematic Risk for Securities and Portfolios: Some Empirical Results. *The Journal of Financial and Quantitative Analysis*, 6(2), 815-833. doi:10.2307/2329716
- Jayasuriya, S. A., & Shambora, W. (2009). Oops, we should have diversified! *Applied Financial Economics*, 19(22), 1779-1785.
- Levy, H., & Sarnat, M. (1970). International diversification of investment portfolios. *The American Economic Review*, 668-675.
- Li, K., Sarkar, A., & Wang, Z. (2003). Diversification benefits of emerging markets subject to portfolio constraints. *Journal of Empirical Finance*, 10(1), 57-80.
- Lim, T., & Ong, C. S. (2021). Portfolio diversification using shape-based clustering. *The Journal of Financial Data Science*, 3(1), 111-126.
- Lin, W.-L., Engle, R. F., & Ito, T. (1994). Do bulls and bears move across borders? International transmission of stock returns and volatility. *Review of Financial Studies*, 7(3), 507-538.
- Longin, F., & Solnik, B. (2001). Extreme correlation of international equity markets. *The journal of finance*, 56(2), 649-676.
- Markowitz, H. (1952). Portfolio selection. *The journal of finance*, 7(1), 77-91.
- Montiel, P. J. (1994). Capital mobility in developing countries: some measurement issues and empirical estimates. *The World Bank Economic Review*, 8(3), 311-350.
- Mostafa, M. Z. P., & Stavroyiannis, S. (2016). BRIC dynamic conditional correlations, portfolio diversification and rebalancing after the global financial crisis of 2008-2009. *Global Business and Economics Review*, 18(1), 28-40.
- Najeeb, S. F., Bacha, O., & Masih, M. (2015). Does Heterogeneity in Investment Horizons Affect Portfolio Diversification? Some Insights Using M-GARCH-DCC and Wavelet Correlation Analysis. *Emerging Markets Finance and Trade*, 51(1), 188-208.
- Newbould, G. D., & Poon, P. S. (1993). The minimum number of stocks needed for diversification. *Financial Practice and Education*, 3(2), 85-87.

- Oloko, T. F. (2018). Portfolio diversification between developed and developing stock markets: The case of US and UK investors in Nigeria. *Research in International Business and Finance*, 45, 219-232.
- Reilly, F., & Brown, K. (2011). *Investment analysis and portfolio management*: Cengage Learning.
- Roy, A. D. (1952). Safety first and the holding of assets. *Econometrica: Journal of the Econometric Society*, 431-449.
- Shahzad, S. J. H., Bouri, E., Roubaud, D., & Kristoufek, L. (2020). Safe haven, hedge and diversification for G7 stock markets: Gold versus bitcoin. *Economic Modelling*, 87, 212-224.
- Statman, M. (1987). How many stocks make a diversified portfolio? *Journal of Financial and Quantitative Analysis*, 22(03), 353-363.
- Sukumaran, A., Gupta, R., & Jithendranathan, T. (2015). Looking at new markets for international diversification: frontier markets. *International Journal of Managerial Finance*, 11(1), 97-116.
- Vo, X. V. (2017). How does the stock market value bank diversification? Evidence from Vietnam. *Finance Research Letters*, 22, 101-104.
- Williams, J. B. (1938). *The theory of investment value* (Vol. 36): JSTOR.